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M2 to D2 revisited

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Abstract: We present two derivations of the multiple D2 action from the multiple M2-brane model proposed by Bagger-Lambert and Gustavsson. The first one is to start from Lie 3-algebra associated with given (arbitrary) Lie algebra. The Lie 3-algebra metric is not positive definite but the zero-norm generators merely correspond to Lagrange multipliers. Following the work of Mukhi and Papageorgakis, we derive D2-brane action from the model by giving a variable a vacuum expectation value. The second derivation is based on the correspondence between M2 and M5. We compactify one dimension and wind M5-brane along this direction. This leads to a noncommutative D4 action. Multiple D2 action is then obtained by suitably choosing the non-commutative parameter on the two-torus. It also implies a natural interpretation to the extra generator in Lie 3-algebra, namely the winding of M5 world volume around S^1 which defines the reduction of M theory to IIA superstring.

Keywords: M-Theory, D-branes.

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1. Introduction

The multiple M2-brane model of Bagger-Lambert [1 – 3] and Gustavsson [4, 5] is defined on Lie 3-algebras [6], which serve as the gauge symmetry algebras for the M2-brane world-volume theory. For the consistency of these symmetries, we need to impose the fundamental identity on the Lie 3-algberas. But it turns out that the fundamental identities are extremely restrictive. For quite some time the only known non-trivial example of Lie 3-algebras is the algebra $\mathcal{A}_4$ [7] with 4 generators and SO(4) symmetry, until many more examples were given in [8]. In fact, Nambu-Poisson brackets [9 – 13] can be viewed as infinite dimensional Lie 3-algebras, and it can be used [14] to construct an M5-brane out of infinitely many M2-branes.

While it is easy to find Nambu-Poisson brackets equipped with positive definite invariant metrics, all finite-dimensional examples, except direct sums of $\mathcal{A}_4$ and trivial algebras, have the salient feature that the invariant metric is never positive definite. It was thus conjectured in [8] (see also [15, 16]) that there exists no other finite dimensional Lie 3-algebras with a positive definite metric. This conjecture was later proved in refs. [17, 18].[1]

While $\mathcal{A}_4$ corresponds to a certain fixed configuration of M2-branes in an M-fold [19 – 22], other Lie 3-algebras are needed for other backgrounds. Thus we either dismiss the BLG model, or we have to accept Lie 3-algebras with zero-norm or negative-norm generators. Some may worry that the existence of negative-norm generators in the Lie 3-algebra may lead to ghosts in the BLG model. Thus a crucial test of the BLG model is whether it

[1] On the other hand, it was suggested [22] that the BLG model is to be studied only at the level of equations of motion, which does not require the definition of an invariant metric. For other interesting development on the multiple M2 theory, see for example [23], in addition to [8, 16 – 19].

can make sense for a Lie 3-algebra with a metric which is not positive definite. Another important task is to find Lie 3-algebras which will lead to $\mathrm{U}(N)$ gauge theories for arbitrary N, in order to describe the configuration of N D2-branes when one of the spatial dimensions is compactified.

In this paper, we first construct a Lie 3-algebra as an extension of an arbitrary Lie algebra (section 2). We show that the BLG model based on this new example of Lie 3-algebra is parity invariant, and the zero-norm generator corresponds to Lagrange multipliers (section 3). Remarkably, the overall coefficient of the Lagrangian has the scaling symmetry, and thus there is no free parameter in this theory. However, we also comment (section 4) that in general one can treat the field components corresponding to certain particular generators as non-dyanmical parameters without breaking supersymmetry or gauge symmetry. This new interpretation completely removes the ghost for our Lie 3-algebra. Following Mukhi and Papageorgakis [19], we consider the reduction of M2 to D2-branes (section 5). There is no ghost after compactification, and a spatial dimension completely disappears, reducing the spacetime dimension from 11 to 10. We find that there are no higher order terms in the D2-brane action, and the translation symmetry is manifestly preserved.

In this approach of deriving multiple D2-branes from M2-branes through a finite dimensional Lie 3-algebra, the physical meaning of the extra generators are not very clear. In section 6, we present the second derivation of D2 from M2. It is based on the construction of M5-brane from M2 [14], where the infinite dimensional version of the Lie 3-algebra based the Nambu-Poisson bracket on three dimensional space was used. It was shown that the field content of BLG theory is mapped to those on M5-brane which include the self-dual two-form field. We compactify one dimension in this internal 3 dimensional manifold and wind one direction of M5-brane along this direction. We compute the BL Lagrangian in this set-up and show that it gives rise to non-commutative D4-brane action where the non-commutativity is infinitesimal. We show that it is possible to generalize the algebra of Nambu-Poisson bracket by quantization to finite non-commutativity. When the internal space is T^2, by suitably choosing the non-commutativity parameter, one may obtain $\mathrm{U}(N)$ symmetry on the D2-brane world volume. In this approach, there is no problem of positivity of the norm from the beginning and it also provides a natural interpretation of one of the extra generators as the winding mode of M5-brane worldvolume.

2. Lie 3-algebra from Lie algebra

For any given Lie algebra $\mathcal{G}$

$$[T^i, T^j] = f^{ij}{}_k T^k \tag{2.1}$$

with structure constants $f^{ij}{}_k$ and Killing form h^{ij}, we can define a corresponding Lie 3-algebra as follows. Let the generators of the Lie 3-algebra be denoted $\{T^{-1}, T^0, T^i\}$ ($i = 1, \cdots, \dim \mathcal{G}$), where T^i's are one-to-one corresponding to the generators of the Lie algebra $\mathcal{G}$. The Nambu bracket is defined by

$$[T^{-1}, T^a, T^b] = 0, \tag{2.2}$$
$$[T^0, T^i, T^j] = f^{ij}{}_k T^k, \tag{2.3}$$
$$[T^i, T^j, T^k] = f^{ijk} T^{-1}, \tag{2.4}$$

where $a, b = -1, 0, 1, \cdots, \dim \mathcal{G}$, and

$$f^{ijk} \equiv f^{ij}{}_l h^{lk} \tag{2.5}$$

is totally anti-symmetrized.

One can check that the Nambu bracket, which is by definition skew-symmetric, satisfies all fundamental identities, that is, for all a, b, c, d, e,

$$[T^a, T^b, [T^c, T^d, T^e]] = [[T^a, T^b, T^c], T^d, T^e] + [T^c, [T^a, T^b, T^d], T^e] + [T^c, T^d, [T^a, T^b, T^e]]. \tag{2.6}$$

The requirement of invariance of the metric

$$\langle [T^a, T^b, T^c], T^d \rangle + \langle [T^c, [T^a, T^b, T^d] \rangle = 0 \tag{2.7}$$

implies that the metric has to be defined as

$$\langle T^{-1}, T^{-1} \rangle = 0, \qquad \langle T^{-1}, T^0 \rangle = -1, \qquad \langle T^{-1}, T^i \rangle = 0, \tag{2.8}$$
$$\langle T^0, T^0 \rangle = K, \qquad \langle T^0, T^i \rangle = 0, \tag{2.9}$$
$$\langle T^i, T^j \rangle = h^{ij}, \tag{2.10}$$

where K is an arbitrary constant and $i, j = 1, \cdots, \dim \mathcal{G}$.

Note that there is an algebra homomorphism

$$T^0 \to T^0 + \alpha T^{-1}, \tag{2.11}$$

that preserves the 3-algebra, but changes the metric by a shift of K:

$$K = \langle T^0, T^0 \rangle \to K - 2\alpha. \tag{2.12}$$

Thus one can always choose T^0 such that

$$K = 0. \tag{2.13}$$

This Lie 3-algebra has the following interesting properties.

1. The Lie 3-algebra reduces to the Lie algebra when one of the slots of the Nambu bracket is taken by T^0. That is,

$$[T^0, T^i, T^j] = [T^i, T^j], \tag{2.14}$$

where the bracket on the right hand side is the Lie algebra bracket.

2. The generator T^0 never appears on the right hand side of a Nambu bracket.

3. The generator T^{-1} is central, that is, the Nambu bracket vanishes whenever T^{-1} appears.

4. There are negative-norm generators. The norm of $T^0 + \alpha T^1$ is $K - 2\alpha$, which is negative for sufficiently large α. T^1 is a zero-norm generator.

5. Generally speaking, the scaling of structure constants

$$f^{abc}{}_d \to g^2 f^{abc}{}_d \tag{2.15}$$

defines a new Lie 3-algebra, since the scaled structure constants must also satisfy all the fundamental identities. We can scale the generators $T^a \to g T^a$ to absorb this scaling, so that the structure constants are scaled back to their original values, but this will result in a scaling of the metric $h^{ab} \to g^2 h^{ab}$. However, for the particular Lie 3-algebra under investigation, a scaling of the structure constants (2.15) can be absorbed by the scaling

$$T^0 \to g^2 T^0, \quad T^{-1} \to g^{-2} T^{-1}, \quad T^i \to T^i, \tag{2.16}$$

which does not change the metric at all.

These properties will be important for the consideration of multiple M2-branes.

3. Bagger-Lambert lagrangian

In this section we apply the Lie 3-algebra constructed in the previous section to the Bagger-Lambert action [1−3], which is a supersymmetric action proposed to describe multiple M2-branes:

$$S = T_2 \int d^3x \, \mathcal{L}, \tag{3.1}$$

where T_2 is the M2-brane tension, and the Lagrangian density $\mathcal{L}$ is

$$\mathcal{L} = -\frac{1}{2}\langle D^\mu X^I, D_\mu X^I\rangle + \frac{i}{2}\langle \bar{\Psi}, \Gamma^\mu D_\mu \Psi\rangle + \frac{i}{4}\langle \bar{\Psi}, \Gamma_{IJ}[X^I, X^J, \Psi]\rangle - V(X) + \mathcal{L}_{\rm CS}. \tag{3.2}$$

Here D_μ is the covariant derivative

$$(D_\mu X^I(x))_a = \partial_\mu X^I_a - f^{cdb}{}_a A_{\mu cd}(x) X^I_b, \tag{3.3}$$

$V(X)$ is the potential term defined by

$$V(X) = \frac{1}{12}\langle [X^I, X^J, X^K], [X^I, X^J, X^K]\rangle, \tag{3.4}$$

and the Chern-Simons term for the gauge potential is

$$\mathcal{L}_{\rm CS} = \frac{1}{2}\epsilon^{\mu\nu\lambda}\left(f^{abcd} A_{\mu ab}\partial_\nu A_{\lambda cd} + \frac{2}{3} f^{cda}{}_g f^{efgb} A_{\mu ab} A_{\nu cd} A_{\lambda ef}\right). \tag{3.5}$$

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The indices $I, J, K = 3, \cdots, 10$, and they specify the transverse directions of M2-branes; $\mu, \nu = 0, 1, 2$, describing the longitudinal directions. The indices a, b, c take values in $-1, 0, 1, \cdots, \dim \mathcal{G}$ for our Lie 3-algebra introduced in the previous section.

The mode expansions of the fields are

$$
\begin{aligned}
X^I &\equiv X^I_a T^a = X^I_0 T^0 + X^I_{-1} T^{-1} + \hat{X}^I, && (3.6)\\
\Psi &\equiv \Psi_a T^a = \Psi_0 T^0 + \Psi_{-1} T^{-1} + \hat{\Psi}, && (3.7)\\
A_\mu &\equiv A_{\mu ab} T^a \otimes T^b \\
&= T^{-1} \otimes A_{\mu(-1)} - A_{\mu(-1)} \otimes T^{-1} + T^0 \otimes \hat{A}_\mu - \hat{A}_\mu \otimes T^0 + A_{\mu ij} T^i \otimes T^j, && (3.8)
\end{aligned}
$$

where

$$
\begin{aligned}
\hat{X} &\equiv X_i T^i, & \hat{\Psi} &\equiv \Psi_i T^i, && (3.9)\\
A_{\mu(-1)} &\equiv A_{\mu(-1)a} T^a, & \hat{A}_\mu &\equiv 2A_{\mu 0i} T^i. && (3.10)
\end{aligned}
$$

We also define

$$
A'_\mu \equiv A_{\mu ij} f^{ij}{}_k T^k. \tag{3.11}
$$

We will see below that $A_{\mu(-1)}$ are completely decoupled in the BLG model, and X^I_{-1} and Ψ_{-1} are Lagrange multipliers.

The action has $N = 8$ maximal SUSY in $d = 3$, and the SUSY transformations are

$$
\begin{aligned}
\delta X^I_a &= i\bar{\epsilon}\Gamma^I \Psi_a, && (3.12)\\
\delta\Psi_a &= D_\mu X^I_a \Gamma^\mu \Gamma^I \epsilon - \frac{1}{6} X^I_b X^J_c X^K_d f^{bcd}{}_a \Gamma^{IJK}\epsilon, && (3.13)\\
\delta\tilde{A}_\mu{}^b{}_a &= i\bar{\epsilon}\Gamma_\mu \Gamma_I X^I_c \Psi_d f^{cdb}{}_a, \qquad \tilde{A}_\mu{}^b{}_a \equiv A_{\mu cd} f^{cdb}{}_a. && (3.14)
\end{aligned}
$$

In terms of the modes, we have

$$
\begin{aligned}
\delta X^I_0 &= i\bar{\epsilon}\Gamma^I \Psi_0, && (3.15)\\
\delta X^I_{-1} &= i\bar{\epsilon}\Gamma^I \Psi_{-1}, && (3.16)\\
\delta \hat{X}^I &= i\bar{\epsilon}\Gamma^I \hat{\Psi}, && (3.17)\\
\delta\Psi_0 &= \partial_\mu X^I_0 \Gamma^\mu \Gamma^I \epsilon, && (3.18)\\
\delta\Psi_{-1} &= (\partial_\mu X^I_{-1} - \langle A'_\mu X^I \rangle)\Gamma^\mu \Gamma^I \epsilon - \frac{1}{3}\langle \hat{X}^I \hat{X}^J \hat{X}^K \rangle \Gamma^{IJK}\epsilon, && (3.19)\\
\delta\hat{\Psi} &= \hat{D}_\mu \hat{X}^I \Gamma^\mu \Gamma^I \epsilon - \frac{1}{2} X^I_0 [\hat{X}^J, \hat{X}^K]\Gamma^{IJK}\epsilon, && (3.20)\\
\delta\hat{A}_\mu &= i\bar{\epsilon}\Gamma_\mu \Gamma_I (X^I_0 \hat{\Psi} - \hat{X}^I \Psi_0), && (3.21)\\
\delta A'_\mu &= i\bar{\epsilon}\Gamma_\mu \Gamma_I [\hat{X}^I, \hat{\Psi}]. && (3.22)
\end{aligned}
$$

The gauge symmetry for the bosonic fields are written as,

$$
\delta X^I_a = \Lambda_{cd} f^{cdb}{}_a X^I_b\,, \quad \delta\tilde{A}_\mu{}^b{}_a = \partial_\mu \tilde{\Lambda}^b{}_a - \tilde{\Lambda}^b{}_c \tilde{A}_\mu{}^c{}_a + \tilde{A}_\mu{}^b{}_c \tilde{\Lambda}^c{}_a\,. \tag{3.23}
$$

(The gauge transformation of Ψ is the same as X^I.) In terms of the mode expansions, they are

$$\delta X_0^I = 0, \tag{3.24}$$
$$\delta X_{-1}^I = \langle \Lambda', \hat{X}^I \rangle, \tag{3.25}$$
$$\delta \hat{X}^I = [\hat{\Lambda}, \hat{X}^I], \tag{3.26}$$
$$\delta \hat{A}_\mu = \partial_\mu \hat{\Lambda} - [\hat{A}_\mu, \hat{\Lambda}], \tag{3.27}$$
$$\delta A'_\mu = \partial_\mu \Lambda' - [\hat{A}_\mu, \Lambda'] - [A'_\mu, \hat{\Lambda}], \tag{3.28}$$

where

$$\hat{\Lambda} = 2\Lambda_{0i} T^i, \qquad \Lambda' = \Lambda_{ij} f^{ij}{}_k T^k. \tag{3.29}$$

Plugging the mode expansions (3.6)–(3.8) into the Lagrangian (3.2), we get, up to total derivatives,

$$\mathcal{L} = \Big\langle -\frac{1}{2}(\hat{D}_\mu \hat{X}^I - A'_\mu X_0^I)^2 + \frac{i}{4}\bar{\hat{\Psi}}\Gamma^\mu \hat{D}_\mu \hat{\Psi} + \frac{i}{4}\bar{\Psi}_0 \Gamma^\mu A'_\mu \hat{\Psi} + \frac{1}{4}(X_0^K)^2 [\hat{X}^I, \hat{X}^J]^2 \tag{3.30}$$
$$- \frac{1}{2}(X_0^I [\hat{X}^I, \hat{X}^J])^2 + \frac{1}{2}\epsilon^{\mu\nu\lambda} \hat{F}_{\mu\nu} A'_\lambda \Big\rangle + \mathcal{L}_{\rm gh},$$

where

$$\mathcal{L}_{\rm gh} \equiv - \Big\langle \partial_\mu X_0^I A'_\mu \hat{X}^I + (\partial_\mu X_0^I)(\partial_\mu X_{-1}^I) - \frac{i}{2}\bar{\Psi}_{-1}\Gamma^\mu \partial_\mu \Psi_0 \Big\rangle, \tag{3.31}$$

and

$$\hat{D}_\mu X^I \equiv \partial_\mu \hat{X}^I - [\hat{A}_\mu, \hat{X}^I], \qquad \hat{D}_\mu \Psi \equiv \partial_\mu \hat{\Psi} - [\hat{A}_\mu, \hat{\Psi}], \qquad \hat{F}_{\mu\nu} \equiv \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu - [\hat{A}_\mu, \hat{A}_\nu]. \tag{3.32}$$

This Lagrangian is invariant under the parity transformation

$$x^\mu \to -x^\mu, \qquad \Gamma^\mu \to -\Gamma^\mu, \tag{3.33}$$
$$\hat{X}^I \to \hat{X}^I, \qquad X_0^I \to -X_0^I, \qquad X_{-1}^I \to -X_{-1}^I, \tag{3.34}$$
$$\hat{\Psi} \to \hat{\Psi}, \qquad \Psi_0 \to -\Psi_0, \qquad \Psi_{-1} \to -\Psi_{-1}, \tag{3.35}$$
$$\hat{A}_\mu \to -\hat{A}_\mu, \qquad A'_\mu \to A'_\mu. \tag{3.36}$$

Another symmetry of this model is the scaling transformation of the overall coefficient of the Lagrangian. Usually a scaling of the structure constants is equivalent to a scaling of the overall constant factor of the action through a scaling of all fields. This overall factor is then an unfixed coupling, which is undesirable in M theory. However, the situation is different for our new algebra. As we commented in the previous section, the scaling of structure constants for the new algebra can be absorbed by a scaling of T^0 and T^{-1} without changing the metric. In other words, the scaling of the overall coefficient of the Lagrangian is a symmetry. Explicitly, scaling (3.30) by an overall coefficient $1/g^2$ can be absorbed by the field redefinition

$$\hat{X}^I \to g\hat{X}^I, \qquad X_0^I \to g^{-1} X_0^I, \qquad X_{-1}^I \to g^3 X_{-1}^I, \tag{3.37}$$
$$\hat{\Psi} \to g\hat{\Psi}, \qquad \Psi_0 \to g^{-1}\Psi_0, \qquad \Psi_{-1} \to g^3 \Psi_{-1}, \tag{3.38}$$
$$\hat{A}_\mu \to \hat{A}_\mu, \qquad A'_\mu \to g^2 A'_\mu. \tag{3.39}$$

Hence this Lagrangian has no free parameter at all!

Note also that X^I_{-1} and Ψ_{-1} appear only linearly in L_{-1}, and thus they are Lagrange multipliers. Their equations of motion are

$$\partial^2 X^I_0 = 0, \qquad \Gamma^\mu \partial_\mu \Psi_0 = 0. \tag{3.40}$$

Hence X^I_0 and Ψ_0 become classical fields, in the sense that off-shell fluctuations are excluded from the path integral. Actually we can set

$$X^I_0 = \text{constant}, \quad \Psi_0 = 0, \tag{3.41}$$

without breaking the supersymmetry (3.15)–(3.22) nor gauge symmetry (3.15)–(3.22).

After we set (3.41), the Lagrangian is given by (3.30) without the last term L_{gh}. It is remarkable that the ghost degrees of freedom associated with X^I_{-1} and Ψ_{-1} have totally disappeared for this background. The resulting theory is clearly a well defined field theory without ghosts.

The fact that the background (3.41) does not break any symmetry suggests an alternative viewpoint towards the BLG model. That is, we can change the definition of the BLG model by defining X^I_0, Ψ_0 as non-dynamical constant parameters fixed by (3.41). The resulting model has as large symmetry as the original definition of the BLG model, but has no ghosts. In this interpretation, the parameter X^I_0 plays the role of coupling constant.

4. Reduction of 3-algebras in BLG model

From the example of the new 3-algebra described above, we see that in general there are two kinds of 3-algebra generators that are special from the viewpoint of the BLG model.

First, if a generator T^A can never be generated through a Nambu bracket (like T^0 in our 3-algebra), i.e.

$$f^{abc}{}_A = 0 \quad \forall a, b, c, \tag{4.1}$$

then $\tilde{A}_\mu{}^b{}_A = 0$, and it is straightforward to check that for the assignment

$$X^I_A = \text{constant}, \quad \Psi_A = 0 \tag{4.2}$$

on the components corresponding to this generator T^A, we have $D_\mu X^I_A = 0$ and the SUSY transformations of the fixed components vanish

$$\delta X^I_A = \delta \Psi_A = 0 \tag{4.3}$$

for arbitrary SUSY transformation parameter ϵ. Thus the complete SUSY is preserved by (4.2).

For the gauge symmetry, if we define the gauge transformation parameter in (3.23) as

$$\tilde{\Lambda}^b{}_a = \Lambda_{cd} f^{cdb}{}_a, \tag{4.4}$$

then for arbitrary Λ_{cd}, we have all gauge transformations of the fixed components vanish. Hence the gauge symmetry is preserved for arbitrary Λ_{cd}. However, there is the possibility

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that in some cases not all degrees of freedom in $\tilde{\Lambda}^b{}_a$ correspond to Λ_{cd}, and the corresponding gauge symmetry may be broken, while all those which can be written in terms of Λ_{cd} are preserved.

Similarly, if a generator T^A is central (like T^{-1} in our 3-algebra), i.e.,

$$f^{Aab}{}_c = 0 \quad \forall a, b, c, \tag{4.5}$$

then the assignment

$$X^{IA} = \text{constant}, \quad \Psi^A = 0 \tag{4.6}$$

preserves SUSY and gauge symmetry. Here the index A is raised using the invariant metric

$$X^{IA} \equiv X^I_a h^{aA}, \quad \text{etc.} \tag{4.7}$$

Furthermore, corresponding to the central element T^A, the components

$$X^I_A, \quad \Psi_A, \quad \tilde{A}_\mu{}^b{}_A \tag{4.8}$$

cannot appear in the interaction terms. X^I_A and Ψ_A can only appear in the kinetic terms, while $\tilde{A}_\mu{}^b_A$ is completely decoupled.

Since the metric components for central elements are not constrained by the requirement of invariance, we can always choose them to vanish

$$h^{AB} = 0, \tag{4.9}$$

and the components X^I_A and Ψ_A can only appear linearly in the kinetic terms. They can then be integrated out as Lagrange multipliers.

As the assignments (4.2) and (4.6) for two special types of generators preserve all SUSY and gauge symmetries, one can take the viewpoint that these variables are non-dynamical *by definition*. We have seen earlier that this interpretation removes the ghost from the BLG model for our new 3-algebra.

5. From M2 to D2

Let us now consider the theory defined in section 3 for the particular background

$$X^I_0 = v^I, \qquad \Psi_0 = 0, \tag{5.1}$$

where v is a constant vector. Without loss of generality, for space-like vector v, we can choose v to lie on the direction of X^{10}

$$v^I = v\, \delta^I_{10}. \tag{5.2}$$

As we mentioned in the previous section, fixing the fields X^I_0 and Ψ_0 by (5.1) removes the ghost term $\mathcal{L}_{\text{gh}}$ from the Lagrangian. We can now integrate over A' and find

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2}(\hat{D}_\mu \hat{X}^A)^2 + \frac{1}{4}v^2[\hat{X}^A, \hat{X}^B]^2 + \frac{i}{4}\bar{\hat{\Psi}}\Gamma^\mu \hat{D}_\mu \hat{\Psi} - \frac{1}{4v^2}\hat{F}^2_{\mu\nu}, \tag{5.3}$$

where $A, B = 3, \cdots, 9$.

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It is very interesting to note that all degrees of freedom in the spatial coordinate X^{10} have totally disappeared from both the kinetic term and the potential term of the action. It is fully decoupled from the Lagrangian for the particular background under consideration.

Let us now recall that when M theory is compactified on a circle, it is equivalent to type IIA superstring theory and M2-branes are matched with D2-branes. The background (5.1) considered above is reminiscent of the novel Higgs mechanism in [19]. It was originally proposed to describe the effect of compactification of X^{10}, and later found to correspond to a large k limit of a $\mathbb{Z}_{2k}$ M-fold [20, 21].

The M theory parameters can be converted to the parameters of type IIA superstring theory via

$$R = g_s l_s, \qquad \text{and} \qquad T_s \equiv \frac{1}{2\pi\alpha'} = 2\pi R T_2. \tag{5.4}$$

The Lagrangian (5.3) is thus exactly the same as the low energy effective action of multiple D2-branes if v is given by the perimeter of the compactified dimension

$$v = 2\pi R. \tag{5.5}$$

Despite the similarity, there are a few features of our model that are different from [19]:

1. The action (5.3) does not have higher order terms.

2. The translation symmetry of the center of mass coordinates corresponding to the $u(1)$ factor of $\mathcal{G}$ is manifest.

These are considered as stronger signatures of the reduction of M2 to D2 due to a compactification of the M theory on S^1.

As the D2-brane is dual to M2-brane, the 11-th dimension of the M theory is not lost when X^{10} disappears. It is dual to the gauge field degrees of freedom on the D2-brane [25].

6. From M5 to D2

In this section, we present a very different derivation of D2-brane from M2. It is based on the derivation of M5-brane from BLG theory [14]. We consider a three dimensional manifold $\mathcal{N}$ equipped with the Nambu-Poisson structure. By choosing the appropriate local coordinates $y^{\dot\mu}$ ($\dot\mu = \dot 1, \dot 2, \dot 3$), one may construct an infinite dimensional Lie 3-algebra from the basis of functions on $\mathcal{N}$, χ^a ($a = 1, 2, 3, \cdots$) as,

$$\left\{\chi^a, \chi^b, \chi^c\right\} = \sum_d f^{abc}{}_d \, \chi^d, \quad \{f_1, f_2, f_3\} = \sum_{\dot\mu,\dot\nu,\dot\lambda} \epsilon_{\dot\mu\dot\nu\dot\lambda} \frac{\partial f_1}{\partial y^{\dot\mu}} \frac{\partial f_2}{\partial y^{\dot\nu}} \frac{\partial f_3}{\partial y^{\dot\lambda}}\,. \tag{6.1}$$

From the property of the Nambu-Poisson structure, this 3-algebra satisfies the fundamental identity with positive definite and invariant metric for the generators,

$$\langle \chi^a, \chi^b \rangle = \int_{\mathcal{N}} d^3y \chi^a(y) \chi^b(y)\,. \tag{6.2}$$

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By the summation of these generators with the fields in BL action,

$$X^I(x,y) = \sum_a X_a^I(x)\chi^a(y)\,, \tag{6.3}$$

$$\Psi(x,y) = \sum_a \Psi_a(x)\chi^a(y)\,, \tag{6.4}$$

$$A_\mu(x,y,y') = \sum_{a,b} A_{\mu ab}(x)\chi^a(y)\chi^b(y'), \tag{6.5}$$

we obtain the fields on the six dimensional manifold $\mathcal{M}\times\mathcal{N}$ where $\mathcal{M}$ is the world volume of the original membrane. We note that the gauge field $A_\mu(x,y,y')$ appears to depends on two points on $\mathcal{N}$. However, if we examine the action carefully, one can show that it depends on $A_\mu(x,y,y')$ only through [24],

$$b_{\mu\dot\nu}(x,y) = \frac{\partial}{\partial y'^{\dot\nu}} A_\mu(x,y,y')\bigg|_{y'=y}\,. \tag{6.6}$$

Therefore the action can be written in terms of the local fields. It was shown that the BL Lagrangian, after suitable field redefinitions, describes the field theory on M5 [14] which includes the self-dual two-form field. While the analysis in [14] is at the level of quadratic order, we will present here the nonlinear action which includes all the terms in BL action. This is based on a technical development in [24] where the exact analysis including the nonlinear terms are given. Because the full detail of the computation is given in [24], we present only the result and its implication here.

In order to obtain D4 from M2, we have to wind $X^{\dot3}$ around the compact $y^{\dot3}$ direction [26] and impose the constraints that the other fields do not depend on $y^{\dot3}$. Other than that, we use the same field configuration [14]

$$X^{\dot3} = y^{\dot3}\,, \tag{6.7}$$

$$X^{\dot\alpha} = y^{\dot\alpha} + \epsilon_{\dot\alpha\dot\beta}a_{\dot\beta}(x,y)\,, \tag{6.8}$$

$$a_\mu(x,y) = b_{\mu\dot3}(x,y)\,, \tag{6.9}$$

$$\tilde a_\lambda(x,y) = \epsilon_{\dot\alpha\dot\beta}\partial_{\dot\alpha}b_{\lambda\dot\beta}\,, \tag{6.10}$$

$$\partial_{\dot3}X^i = \partial_{\dot3}\Psi = \partial_{\dot3}a_{\dot\beta} = \partial_{\dot3}a_\mu = \partial_{\dot3}\tilde a_\lambda = 0\,. \tag{6.11}$$

Here we use the indices $\dot\alpha,\dot\beta,\cdots$ to denote $\dot1,\dot2$ such that the world volume index of D4 is μ and $\dot\alpha$. We use the notation $i=1,\cdots,5$ for the transverse directions. We repeat the same computation as in [14] but here we include the nonlinear terms. It turns out that $b_{\mu\dot\nu}$ appears only through a_μ and $\tilde a_\mu$.

Various terms of the D4 action can be computed [14, 24] straightforwardly. First the potential term becomes

$$-\frac{1}{12}\langle[X^I,X^J,X^K]^2\rangle = \int_{\mathcal{N}} d^3y\left(-\frac12 - F_{\dot1\dot2} - \frac14 F_{\dot\alpha\dot\beta}{}^2 - \frac14\mathcal{D}_{\dot\alpha}X_i{}^2 - \frac14\left\{X_i,X_j\right\}^2\right), \tag{6.12}$$

where

$$F_{\dot\alpha\dot\beta} := \partial_{\dot\alpha}a_{\dot\beta} - \partial_{\dot\beta}a_{\dot\alpha} + \left\{a_{\dot\alpha},a_{\dot\beta}\right\}\,,\quad \mathcal{D}_{\dot\alpha}X_i = \partial_{\dot\alpha}X_i + \{a_{\dot\alpha},X_i\}\,. \tag{6.13}$$

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While we expect to have the Abelian U(1) gauge field on the world volume, we have the Poisson bracket

$$\{f,g\} = \sum_{\dot\alpha,\dot\beta=\dot 1,\dot 2} \epsilon_{\dot\alpha\dot\beta}\partial_{\dot\alpha} f \partial_{\dot\beta} g \tag{6.14}$$

everywhere. It implies that we can not escape from the noncommutativity in $\mathcal{N}$ direction as long as we start from BL Lagrangian. The Chern-Simons term (3.5) becomes, after partial integrations,

$$\mathcal{L}_{\rm CS} = -\frac{1}{2}\epsilon^{\mu\nu\lambda}\int d^3y\, \tilde a_\mu(x,y) F_{\nu\lambda}(x,y)\,, \quad F_{\mu\nu} := \partial_\mu a_\nu - \partial_\nu a_\mu + \{a_\mu, a_\nu\}\,. \tag{6.15}$$

Finally the kinetic terms for X^I and the fermion become

$$-\frac{1}{2}\langle (D_\mu X^I)^2\rangle = -\frac{1}{2}\int_{\mathcal{N}} d^3y \left(F_{\mu\dot\alpha}{}^2 + \tilde a_\mu^2 + \mathcal{D}_\mu X_i{}^2\right)\,, \tag{6.16}$$

$$\frac{i}{2}\langle\bar\Psi,\Gamma^\mu D_\mu\Psi\rangle + \frac{i}{4}\langle\bar\Psi,\Gamma_{IJ}[X^I,X^J,\Psi]\rangle = \frac{i}{2}\int_{\mathcal{N}} d^3y \left(\,\bar\Psi\Gamma^\mu\mathcal{D}_\mu\Psi + \bar\Psi\Gamma^{\dot\alpha}\mathcal{D}_{\dot\alpha}\Psi + \bar\Psi\Gamma_i\left\{X^i,\Psi\right\}\right)\,,$$

where

$$F_{\mu\dot\alpha} := \partial_\mu a_{\dot\alpha} - \partial_{\dot\alpha} a_\mu + \{a_\mu, a_{\dot\alpha}\}\,, \qquad \mathcal{D}_\mu X_i = \partial_\mu + \{a_\mu, X_i\}\,, \tag{6.17}$$

$$\mathcal{D}_\mu\Psi = \partial_\mu\Psi + \{a_\mu,\Psi\}\,, \qquad \mathcal{D}_{\dot\alpha}\Psi = \partial_{\dot\alpha}\Psi + \{a_{\dot\alpha},\Psi\}\,, \tag{6.18}$$

$$\Gamma_{\dot\alpha} = \sum_{\dot\beta}\Gamma_{\dot 3\dot\beta}\epsilon_{\dot\beta\dot\alpha}\,, \qquad \Gamma_i = \Gamma_{\dot 3 i}\,. \tag{6.19}$$

We note that the field $\tilde a_\mu$ does not have the kinetic term and can be integrated out exactly. The integrand does not depend on $y^{\dot 3}$ so we obtain overall factor of $2\pi R$ (R is the radius of the compactified direction) after the integration over $y^{\dot 3}$.

We note that in the computation, there are no ambiguities associated with the inner product. After integrating out the auxiliary field $\tilde a_\mu$, one arrives at the D4-brane action (after neglecting the constant term and the total derivative term)

$$S = 2\pi R\int d^5x \left(-\frac{1}{4}F_{\underline{\mu}\underline{\nu}}{}^2 - \frac{1}{2}\mathcal{D}_{\underline{\mu}}X^{i2} + \frac{i}{2}\bar\Psi\Gamma^{\underline{\mu}}\mathcal{D}_{\underline{\mu}}\Psi - \frac{1}{4}\left\{X^i,X^j\right\}^2 + \frac{i}{2}\bar\Psi\Gamma_i\left\{X^i,\Psi\right\}\right)\,. \tag{6.20}$$

Here $\underline{\mu},\underline{\nu},\cdots$ are the integrated indices for μ,ν and $\dot\alpha,\dot\beta$ run from 0 to 4. As already mentioned, $A_{\underline{\mu}} = a_\mu, a_{\dot\alpha}$ is not exactly the commutative U(1) gauge field but it includes noncommutativity in $\underline{\mu} = 3,4$ directions (originally $\dot\alpha$ directions). The definition of the field strength and the covariant derivatives are, of course,

$$F_{\underline{\mu}\underline{\nu}} = \partial_{\underline{\mu}}A_{\underline{\nu}} - \partial_{\underline{\nu}}A_{\underline{\mu}} + \left\{A_{\underline{\mu}}, A_{\underline{\nu}}\right\}\,,$$

$$\mathcal{D}_{\underline{\mu}}X^i = \partial_{\underline{\mu}}X^i + \left\{A_{\underline{\mu}}, X^i\right\}\,, \qquad \mathcal{D}_{\underline{\mu}}\Psi = \partial_{\underline{\mu}}X^i + \left\{A_{\underline{\mu}},\Psi\right\}\,. \tag{6.21}$$

The origin of the noncommutativity is obvious. It comes from the Nambu-Poisson bracket where the space of the function is truncated to

$$\left\{y^3\right\}\cup C(\mathcal{N}')\,. \tag{6.22}$$

Here we decompose $\mathcal{N}$ into $y^{\dot{3}}$ direction and $\mathcal{N}'$ described by $y^{\dot{1},\dot{2}}$. The Nambu-Poisson bracket becomes (for $f_i(y^{\dot{1}}, y^{\dot{2}}) \in C(\mathcal{N}')$)

$$\left\{y^{\dot{3}}, f_1, f_2\right\}^{NP} = \{f_1, f_2\}\ , \quad \{f_1, f_2, f_3\}^{NP} = 0\,, \quad \text{others} = 0\,. \tag{6.23}$$

The commutator terms in the lagrangian come from this algebra. This algebra turns out to be identical to Lie 3-algebra (2.2–2.4) if we put T^{-1} to zero. The generator that corresponds to T^0 is $y^{\dot{3}}$, which describes the winding of M5 world volume around S^1.

The Poisson bracket $\{f, g\}$ can be obtained from the matrix algebra when the matrix size N is infinite. By using the standard argument (see for example [27]), it is easy to claim that the D4 action which we just obtained can be regarded as describing an infinite number of D2-branes.

However, in order to obtain the finite N theory on D2-brane, this is not sufficient. We need to quantize the Nambu bracket. In general, the quantum Nambu bracket is very difficult to define. However, for the truncated Hilbert space (6.22), this is actually possible. We deform the Nambu-Poisson bracket by,

$$[f_1, f_2, f_3]^{QN} = \sum_{i,j,k=1}^{3} \epsilon_{ijk}(f_i \star f_j)\partial_3 f_k \tag{6.24}$$

where $\star$ is the Moyal product,

$$(f \star g)(y^{\dot{1}}, y^{\dot{2}}) = \exp(i\epsilon_{\dot{\alpha}\dot{\beta}}\theta\partial_{y^{\dot{\alpha}}}\partial_{z^{\dot{\beta}}})f(y^{\dot{1}}, y^{\dot{2}})g(z^{\dot{1}}, z^{\dot{2}})|_{z=y}\,. \tag{6.25}$$

It does not satisfy the fundamental identity when we consider $C(\mathcal{N})$ as a whole. If we restrict the generators to (6.22), we can recover the fundamental identity. If we take $\mathcal{N}'$ as T^2 and quantize θ suitably, the quantum T^2 reduces to the $\mathrm{U}(N)$ algebra,

$$UV = VU\omega, \quad \omega^N = 1\,, \quad U^N = V^N = 1\,. \tag{6.26}$$

In this case the quantum Nambu-Poisson bracket reduces to the one-generator extension of $\mathrm{U}(N)$ algebra

$$[T^0, T^i, T^j] = f^{ij}{}_k T^k\,, \quad [T^i, T^j, T^k] = 0\,. \tag{6.27}$$

The multiple D2 action can be obtained by expanding the functions in $y^{\dot{1},\dot{2}}$ directions by U, V and replacing the covariant derivative $\mathcal{D}_{\dot{\alpha}}$ by the commutators

$$\mathcal{D}_{\dot{\alpha}}\Phi \to [X_{\dot{\alpha}}, \Phi] \tag{6.28}$$

for general Φ.

In this way, by taking a path M2 $\to$ M5 $\to$ D4 $\to$ D2, one can obtain the multiple D2 theory without touching the problem of the negative-norm state.

7. Conclusion

In this paper, we study two approaches to obtain multiple D2-brane action from the BLG theory. In the first approach, one defines Lie 3-algebra which contains generators of a given Lie algebra. Such an extension inevitably contains generators with negative norms. We argued that by suitably choosing such extension, one might restrict the field associated with it to constant or zero while keeping almost all of the symmetry of BLG theory. Such truncation leads to the symmetry breaking mechanism of [19] and generates the standard kinetic term for the gauge fields on the multiple D2-brane worldvolume.

In [8], we have presented many examples of Lie 3-algebras which satisfy the fundamental identity. The algebra which we consider here is a generalization of one of them. It is quite interesting to conjecture that similar mechanism which we consider here may be applied to other examples by restricting the fields associated with the null/negative norm generators to constants. Such theories may not describe M2 or D2 but would give a new insight into M theory dynamics.

In the second derivation of multiple D2-brane, we found that the extra generator has a simple physical origin, the winding of M5-brane around S^1 which defines the reduction from M theory to the type ⅡA theory. One may provide a similar geometrical origin to other Lie 3-algebras.

We also commented that to have finite N theory from M5, we need quantization of the Nambu-Poisson bracket. This is trivially possible in our case for D4-branes since we have reduced the Namb-Poisson bracket into the usual Poisson bracket. In general, however, we need to consider the quantization of full Nambu-Poisson bracket in the full function space. We hope that the many studies in the past [28, 10, 29] would provide a breakthrough toward this direction.

Note added. When we have almost finished the paper, there appeared a paper [30] which overlaps considerably on the first proposal of this paper for deriving D2 from M2 in the BLG model.

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